1. Einfache Integrale:

a) $\int x dx = \frac{1}{2}x^2 + C$

b) $\int_1^3 x dx = \frac{1}{2}x^2\Big|_1^3 = \frac{9}{2} - \frac{1}{2} = \frac{8}{2} = 4$

c) $\int \cos(x) dx = \sin(x) + C$

d) $\int_{\frac{\pi}{2}}^{\pi} \cos(x) dx = \sin(x)\Big|_{\frac{\pi}{2}}^{\pi} = \sin(\pi) - \sin(\frac{\pi}{2}) = -1$

e) $\int (x + x^2) dx = \frac{1}{2}x^2 + \frac{1}{3}x^3 + C$

f) $\int 4x^5 dx = \frac{4}{6}x^6 + C = \frac{2}{3}x^6 + C$

g) $\int (\sqrt[3]{x} + \cos(x) - x) dx = \frac{3}{4}x^{\frac{4}{3}} + \sin(x) - \frac{1}{2}x^2 + C = \frac{3}{4}\sqrt[3]{x^4} + \sin(x) - \frac{1}{2}x^2 + C$

h) $\int (x + 2) dx = \frac{1}{2}x^2 + 2x + C$

i) $\int_1^2 (x + 2) dx = \frac{1}{2}x^2 + 2x\Big|_1^2 = \frac{4}{2} + 4 - (\frac{1}{2} + 2) = 3{,}5$

j) $\int_a^b \frac{dx}{2x} = \frac{1}{2}\int_a^b \frac{1}{x} dx = \frac{1}{2}\ln(x)\Big|_a^b = \frac{1}{2}\left(\ln(b) - \ln(a)\right) = \frac{1}{2}\ln\left(\frac{b}{a}\right) = \ln\left(\sqrt{\frac{b}{a}}\right)$

k) $\int_0^2 \left(e^x + 8\right) dx = e^x + 8x\Big|_0^2 = e^2 + 16 - (e^0 + 0) = e^2 + 15$

l) $\int \frac{x^2 - 5\sqrt{x} + 2\sqrt[3]{x^2}}{\sqrt[4]{x^5}} dx = \int \left(x^{\frac{3}{4}} - 5x^{\frac{-3}{4}} + 2x^{\frac{-7}{12}} \right) dx = \frac{4}{7}x^{\frac{7}{4}} - 20x^{\frac{1}{4}} + \frac{24}{5}x^{\frac{5}{12}} + C$

2. Unbestimmte Integrale:

a) $\int (x^5 - 3x^2 + 7)dx = \frac{x^6}{6} - x^3 + 7x + C$

b) $\int \frac{x^4 - 5x^3 + 2x}{x^3} dx = \frac{x^2}{2} - 5x - \frac{2}{x} + C$

c) $\int \sqrt[3]{x} dx = \frac{3}{4}\sqrt[3]{x^4} + C = \frac{3}{4} x \cdot \sqrt[3]{x} + C$

d) $\int 7 \cdot \sqrt[3]{x^4} dx = 3 \cdot \sqrt[3]{x^7} + C = 3x^2 \cdot \sqrt[3]{x} + C$

e) $\int \sqrt{u \cdot \sqrt[3]{u^2}} du = \int \sqrt{u \cdot u^{\frac{2}{3}}} du = \int \sqrt{u^{\frac{5}{3}}} du = \int u^{\frac{5}{6}} du = \frac{6}{11} \cdot \sqrt[6]{u^{11}} + C = \frac{6}{11} u \cdot \sqrt[6]{u^5} + C$

f) Gesucht ist die Funktion, die bei x = 2 eine Nullstelle hat und deren Ableitung durch $f(x) = x^2$ dargestellt wird.

Lsg.: $\int x^2 dx = \frac{1}{3} x^3 + C$; $\frac{1}{3}(2)^3 + C = 0$; $C = -\frac{8}{3}$; $F(x) = \frac{1}{3} x^3 - \frac{8}{3}$

g) Gesucht ist die Stammfunktion zu $f(x) = x^3 + 5x - 4$, deren Kurve durch den Punkt P(2;8) geht

Lsg.: $\frac{1}{4} x^4 + \frac{5}{2} x^2 - 4x + 2$

3. Bestimmte Integrale:

a) $\int_0^2 x^3 dx = 4$;

b) $\int_0^4 \sqrt{x} dx = \int_0^4 x^{\frac{1}{2}} dx = \frac{2}{3} x^{\frac{3}{2}} \Big|_0^4 = \frac{2}{3}\sqrt{x^3}\Big|_0^4 = \frac{2}{3}\sqrt{64} = \frac{16}{3}$

c) $\int_1^2 \frac{dx}{x} = \ln(x)\Big|_1^2 = \ln(2) - \ln(1) = \ln(2)$

d) $\int_{r_1}^{r_2} \frac{1}{r} dr = \ln(r)\Big|_{r_1}^{r_2} = \ln(r_2) - \ln(r_1)$

e) $\int_0^4 \frac{dx}{\sqrt{x}} = \int_0^4 x^{-\frac{1}{2}} dx = 2x^{\frac{1}{2}}\Big|_0^4 = 2\sqrt{x}\Big|_0^4 = 4$

f) $\int_0^{\pi} \sin(t) dt = -\cos(t)\Big|_0^{\pi} = 1 - (-1) = 2$

g) $\int_0^{\pi} \cos(t) dt = \sin(t)\Big|_0^{\pi} = 0$

h) $\int_{-2}^{1} (2 - 4x) dx = 2x - 2x^2\Big|_{-2}^{1}$ = 2 - 2 - (-4 - 8) = 12

i) $\int_0^{\pi} (\sin(x) - 5) dx = -\cos(x) - 5x\Big|_0^{\pi} = -\cos(\pi) - 5\pi - (-\cos(0) - 5 \cdot 0) = 1 - 5\pi + 1 = 2 - 5\pi$

j) $\int_1^3 (1 - x^2) dx - \int_3^4 (x^2 - 1) dx = \int_1^3 (1 - x^2) dx + \int_3^4 (1 - x^2) dx = \int_1^4 (1 - x^2) dx = x - \frac{x^3}{3}\Big|_1^4$ = -18