1. Lösen Sie folgende Integrale:

a) $\int \sqrt{x}dx = \int x^{\frac{1}{2}}dx = \frac{2}{3}x^{\frac{3}{2}} + C = \frac{2}{3}\sqrt{x^3} + C$

b) $\int 2\sin(x)dx = -2\cos(x) + C$

c) $\int 4\sqrt[3]{x^4}dx = 4\int x^{\frac{4}{3}}dx = 4\cdot\frac{3}{7}x^{\frac{7}{3}} + C = \frac{12}{7}\sqrt[3]{x^7} + C$

d) $\int \frac{x^3+x}{5x^3}dx = \int\left(\frac{1}{5}+\frac{1}{5x^2}\right)dx = \int\left(\frac{1}{5}+\frac{x^{-2}}{5}\right)dx = \frac{1}{5}x - \frac{x^{-1}}{5} + C = \frac{1}{5}x - \frac{1}{5x} + C$

e) $\int \sqrt[3]{x^{-3}}dx = \int x^{\frac{-3}{3}}dx = \int x^{-1}dx = \int\frac{1}{x}dx = \ln(x) + C$

f) $\int \frac{1}{(2x+7)^{-2}}dx = \int (2x+7)^2 dx = \int (4x^2+28x+49)dx = \frac{4}{3}x^3 + 14x^2 + 49x + C$

g) $\int r\cdot\cos(t)dt = r\cdot\sin(t) + C$

h) $\int (u^3+5)du = \frac{1}{4}u^4 + 5u + C$

i) $6\int_4^5 t^{-2}dt = -6t^{-1}\Big|_4^5 = -6\frac{1}{t}\Big|_4^5 = -6\cdot\frac{1}{5} - \left(-6\cdot\frac{1}{4}\right) = 0{,}3$

j) $\int_1^2 \cos(\pi)\sin(t)dt = -\cos(\pi)\cos(t)\Big|_1^2 = -\cos(\pi)\cos(2) - \left(-\cos(\pi)\cos(1)\right) = -0{,}956$

k) $\int_{x1}^{x2} \frac{a}{\sqrt[3]{x^2}}dx = \int_{x1}^{x2} a\cdot x^{\frac{-2}{3}}dx = 3ax^{\frac{1}{3}}\Big|_{x1}^{x2} = 3a\left(\sqrt[3]{x2} - \sqrt[3]{x1}\right)$

l) $\int_0^{\frac{\pi}{2}} \omega\cdot\cos(2\pi)dx = \omega\cdot\cos(2\pi)x\Big|_0^{\frac{\pi}{2}} = \omega\frac{\pi}{2}$

m) $\int_3^1 \cos(\varphi)xdx = \cos(\varphi)\frac{x^2}{2}\Big|_3^1 = \cos(\varphi)\left(\frac{1}{2}-\frac{9}{2}\right) = -4\cos(\varphi)$

n) $\int (2^3 + x^2 - r^x)dx = 8x + \frac{1}{3}x^3 - \frac{r^x}{\ln(r)} + C$

2a) $\int_{0,5}^{2,5} \frac{x^3}{10} dx = \left.\frac{x^4}{40}\right|_{0,5}^{2,5} = \frac{625}{640} - \frac{1}{640} = \frac{624}{640} = \frac{39}{40} = 0{,}975$

2b) Flächen sind immer positiv, darum muss das Integral hier aufgeteilt und mit den Beträgen gerechnet werden.

$$\int_{-2}^{0} x dx + \int_{0}^{4} x dx = \left.\frac{x^2}{2}\right|_{-2}^{0} + \left.\frac{x^2}{2}\right|_{0}^{4} = \left|\frac{0^2}{2} - \frac{(-2)^2}{2}\right| + \left|\frac{4^2}{2} - \frac{(0)^2}{2}\right| = \left|-\frac{4}{2}\right| + \left|\frac{16}{2}\right| = 2 + 8 = 10$$

2c) $\int_{-5}^{7} \left(x^2 - 2x - 35\right) dx = \left.\frac{x^3}{3} - x^2 - 35x\right|_{-5}^{7} = \left|-288\right| = +288$